Supersymmetric black holes and the black hole attractor mechanism

Thibeau Wouters

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Recap: Reissner-Nördstrom BH

The *Reissner-Nördstrom* metric in 4D describes a non-rotating, dyonic (electrically and magnetically charged) black hole:

$$\mathrm{d}s^2 = -F(r)\,\mathrm{d}t^2 + \frac{1}{F(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2\,,$$

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The function $F(r)$ is given by

$$F(r) = 1 - \frac{2MG}{r} + \frac{(q^2+p^2)G}{4\pi r^2}\,,$$

and has two roots $r_\pm$: event horizons.

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and has two roots $r_\pm$: event horizons. The solution has no naked singularities (recall: cosmic censorship conjecture) if $r_\pm \in \mathbb{R}$, or

$$M^2 \geq \frac{q^2+p^2}{4\pi G}\,.$$

If we have equality, the BH is extremal, and there is a single horizon at $r = R_S = MG$.

Recap: Reissner-Nördstrom BH, *cont.*

We *only* consider **extremal** BHs. The metric simplifies:

$$\mathrm{d}s^2 = -\left(1 - \frac{MG}{r}\right)^2 \mathrm{d}t^2 + \left(1 - \frac{MG}{r}\right)^{-2} \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2\,.$$

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We prefer to have the horizon at $r = 0$, so redefine $r \to r - MG$:

Extremal Reissner-Nördstrom BH (horizon at $r = 0$)

$$\mathrm{d}s^2 = -\left(1 + \frac{MG}{r}\right)^{-2} \mathrm{d}t^2 + \left(1 + \frac{MG}{r}\right)^2 \left(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2\right)\,.$$

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Setting the scene

About our model:

- We consider $\mathcal{N} = 2$, $D = 4$ ungauged supergravity, coupled to a single gauge vector multiplet.
- We have two Abelian gauge field strengths $F_{\mu\nu}$, $F'_{\mu\nu}$, with charges (p, q) and (p', q').
- We have one complex scalar z, with $\mathrm{Im}(z) > 0$ (Poincaré plane).

D	SUSY	32	24	20	16	12	8	4
11	M	M						
10	MW	IIA, IIB			I ♡			
9	M	$\mathcal{N}=2$			$\mathcal{N}=1$ ♡			
8	M	$\mathcal{N}=2$			$\mathcal{N}=1$ ♡			
7	S	$\mathcal{N}=4$			$\mathcal{N}=2$ ♡			
6	SW	(2, 2), (3, 1), (4, 0)	(2, 1), (3, 0)		(1, 1) ♡, (2, 0) ◇		(1, 0) ♡, ◇, ♣	
5	S	$\mathcal{N}=8$	$\mathcal{N}=6$		$\mathcal{N}=4$ ♡		$\mathcal{N}=2$ ♡, ♣	
4	M	$\mathcal{N}=8$	$\mathcal{N}=6$	$\mathcal{N}=5$	$\mathcal{N}=4$ ♡	$\mathcal{N}=3$ ♡	$\mathcal{N}=2$ ♡, ♣	$\mathcal{N}=1$ ♡, ♣
		SG	SG	SG	SG/SUSY	SG	SG/SUSY	SG/SUSY

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- We have one complex scalar z, with $\text{Im}(z) > 0$ (Poincaré plane).

An example solution has $\text{Re}(z) = 0$, so $z \equiv ie^{-2\phi}$, with ϕ the dilaton field. Metric ansatz is

$$\mathrm{d}s^2 = -e^{2U(r)}\,\mathrm{d}t^2 + e^{-2U(r)}\left(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2\right) .$$

One gauge field is purely electric, the other purely magnetic: we have $(0, q)$, $(p', 0)$.

The dilaton black hole solution

The full solution is given by:

The dilaton BH

$$e^{-2\phi} = H_1/H_2\,, \quad F = \pm\,\mathrm{d}\left(H_1^{-1}\right) \wedge \mathrm{d}t$$
$$e^{-2U} = H_1 H_2\,, \quad G' = \pm\,\mathrm{d}\left(H_2^{-1}\right) \wedge \mathrm{d}t\,,$$

with harmonic functions

$$H_1 = e^{-\phi_0} + \frac{|q|}{4\pi r}\,, \quad H_2 = e^{\phi_0} + \frac{|p'|}{4\pi r}\,,$$

We have defined

- The dual of F': $G'_{\mu\nu} \equiv -e^{-2\phi}\left({}^*F'\right)_{\mu\nu}$.
- $\phi_0 \equiv \phi(r = +\infty)$: dilaton at infinity (or at $\tau = 0$, $\tau = 1/r$).

The dilaton black hole as attractor

Look at the g_{rr} metric component: from $e^{-2U} = H_1 H_2$, we find

$$g_{rr} = 1 + \frac{e^{-\phi_0}|p'| + e^{\phi_0}|q|}{4\pi r} + \frac{|qp'|}{(4\pi r)^2}\,.$$

Compare with RN BH: M from $1/r$ term (watch out: metric with horizon at $r = 0$)

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Mass dilaton BH

$$M = \frac{1}{8\pi G}\left(e^{-\phi_0}|p'| + e^{\phi_0}|q|\right) .$$

The mass depends on $|p'|, |q|$ and on the dilaton at infinity ϕ_0, as we expect for an extremal BH.

The dilaton black hole as attractor, *cont.*

We can write $e^{-2\phi} = H_1/H_2$ as

$$e^{-2\phi} = \frac{|q| + 4\pi r e^{-\phi_0}}{|p'| + 4\pi r e^{\phi_0}} \quad \xrightarrow{r=0} \quad \left| \frac{q}{p'} \right| .$$

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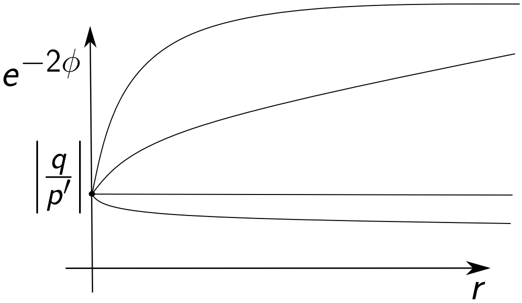
So at the horizon, depends **ONLY** on the charges: any choice of ϕ_0 has the same value of ϕ_{hor}! This is the attractor mechanism. (The term 'attractor' comes from gradient flow equations: see later on) .



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Central charge and BPS bound

In extended susy ($\mathcal{N} > 1$), the algebra admits central charges $\mathcal{Z}$. For $\mathcal{N} = 2$:

$$\{Q_{\alpha i}, Q_{\beta j}\} = -\tfrac{1}{2}\varepsilon_{ij} P_{L\alpha\beta} \mathcal{Z}\,, \quad \{Q_\alpha{}^i, Q_\beta{}^j\} = -\tfrac{1}{2}\varepsilon^{ij} P_{R\alpha\beta} \overline{\mathcal{Z}}\,.$$

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For massive representations in $\mathcal{N} = 2$, we have the BPS bound: $M \geq |\mathcal{Z}|$. When the bound is satisfied ($M = |\mathcal{Z}|$), we have a BPS solution.

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The theory is invariant under susy, but solutions *not necessarily*!
We will use this result from Chapter 12 [1]:

Supersymmetric solutions

A massive solution of sugra is supersymmetric if $M = |\mathcal{Z}|$.
So: a BPS solution is a supersymmetric solution.

Goal: Prove $M = |\mathcal{Z}|$ for the black holes!

The dilaton BH as $\frac{1}{2}$-BPS solution

For $\mathcal{N} = 2, D = 4$ sugra with one vector multiplet, it can be shown that

$$\mathcal{Z} = \frac{1}{8\pi G\sqrt{\mathrm{Im}(z)}}\left[(q + iq') + z(p + ip')\right] \, .$$

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Dilaton BH: charges are $(0, q)$, $(p', 0)$, and $z = ie^{-2\phi}$. Write $q = \pm|q|$ and $p' = \mp|p'|$ (signs related to solution), then:

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Compare with the mass: the BPS bound is satisfied: $M = |\mathcal{Z}_\infty|$: the dilaton BH is supersymmetric! One can show they preserve 1 susy: $\frac{1}{2}$-BPS (in $\mathcal{N} = 2$).

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<u>**BUT:**</u> Eq. (1) is true *only* if signs in $q = \pm|q|, p' = \mp|p'|$ are correlated**!**

- $\mathrm{sgn}(p') = -\mathrm{sgn}(q)$: BPS
- $\mathrm{sgn}(p') = \ \ \mathrm{sgn}(q)$: BPS**?** *Yes*! But need gradient flow eqs.

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Extremal charged black holes are attractors

General case: charges (p, q), (p', q') and $\text{Re}(z) \neq 0$. These are BPS as well! We show this via the attractor mechanism.

The action, using $\mathcal{Z}$, can be written as a sum of squares. The EL equations are easy: require that each square is zero:

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$$\begin{cases} \partial_\tau z & \equiv \dot{z} = -8Ge^U(\mathrm{Im}(z))^2 \partial_{\overline{z}}|\mathcal{Z}| \\ \partial_\tau U & \equiv \dot{U} = -Ge^U|\mathcal{Z}|\,. \end{cases}$$

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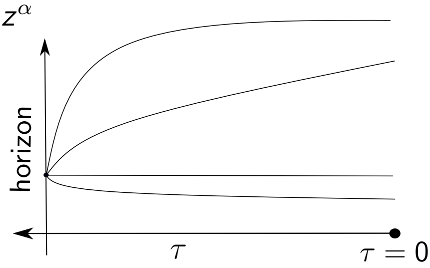
All scalar boundary/initial conditions $(\tau = 0, r = \infty)$ *flow* towards a fixed value $|\mathcal{Z}|_{\mathrm{min}}$: an attractor!

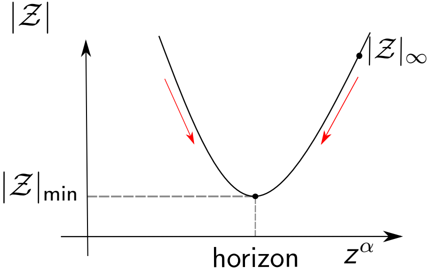
Extremal charged black holes are attractors, *cont.*

Scalar dynamics exhibit a *gradient flow*:

$$\begin{cases} \dot{z} = -8Ge^{U}(\mathrm{Im}(z))^2 \partial_{\bar{z}}|\mathcal{Z}| \\ \dot{U} = -Ge^{U}|\mathcal{Z}| \, . \end{cases}$$

The fixed point is a minimum of $|\mathcal{Z}|$ and lies at the horizon. All scalar initial conditions flow towards a fixed value $|\mathcal{Z}|_{\text{min}}$: an attractor!





Extremal black holes are BPS

The gradient flow allows us to prove $M = |\mathcal{Z}_\infty|$. First, get the mass:

$$-g_{tt} = e^{2U} \approx 1 - 2MG\tau = 1 + \frac{\mathrm{d}\left(e^{2U}\right)}{\mathrm{d}\tau}\bigg|_{\tau=0} \tau \,.$$

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Use the flow equations for $\dot{U}(0)$: this shows $M = |\mathcal{Z}_\infty|$.

Supersymmetric BHs

Extremal charged black hole attractors are supersymmetric solutions of $\mathcal{N} = 2, D = 4$ sugra.

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Supersymmetric BHs

Extremal charged black hole attractors are supersymmetric solutions of $\mathcal{N} = 2, D = 4$ sugra.

Central charges allow us to generalise to couple to n_V gauge vector multiplets (here: $n_V = 1$)

$\rightarrow n_V + 1$ field strengths $F^I_{\mu\nu}$, n_V complex scalars z^α.

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Outlook

(1) Why care about susy BHs?

Cosmic censorship conjecture! Susy can act as a cosmic censor : the BPS bound coincides with the bound that excludes naked singularities [2]. Can easily be shown for the dilaton BH from formulae in this presentation!

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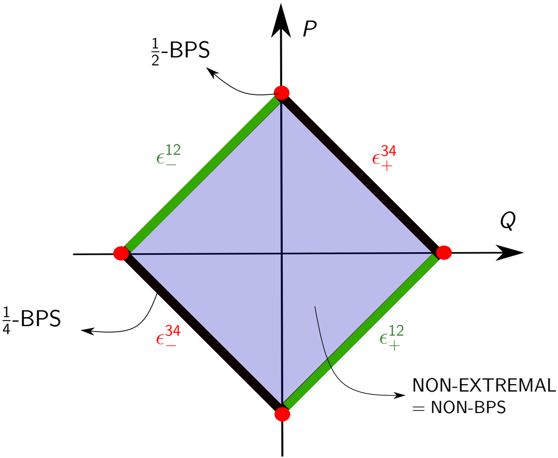
(2) What about generalizations?

In particular, $\mathcal{N} = 4$ sugra. The $\mathcal{N} = 2$ action can be embedded into $\mathcal{N} = 4$. Here, *all* charge configurations arise from Killing spinor conditions. [2].

Diamond of dyonic dilaton black holes in $\mathcal{N} = 4$ sugra

The diamond shows 'censored' BHs, *i.e.* $M \geq |P| + |Q|$. The sides of the square are extremal and BPS, the shaded region are non-BPS. Green solutions are the BPS dilaton BHs found in $\mathcal{N} = 2$ sugra.



Conclusion

- Extremal dyonic black holes are supersymmetric solutions of $\mathcal{N} = 2, D = 4$ supergravity coupled to n_V vector multiplets.
- The scalar fields exhibit a gradient flow towards fixed points at the horizon, which depends on the charges of the black hole, *not* on asymptotic values of scalars (attractor mechanism).
- Central charges from extended susy play an important role (proving the BPS condition, obtaining the gradient flow equations, the area of the horizon).
- Supersymmetry can act as a cosmic censor and hide naked singularities: BPS BHs agree with cosmic censorship.

References & Thanks for listening!

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Cosmic censorship for the dilaton BH

Look at the g_{rr} metric component: from $e^{-2U} = H_1 H_2$, we find

$$g_{rr} = 1 + \frac{e^{-\phi_0}|p'| + e^{\phi_0}|q|}{4\pi r} + \frac{|qp'|}{(4\pi r)^2} .$$

Compare with RN BH:

$$g_{rr} = 1 + \frac{2MG}{r} + \frac{(MG)^2}{r^2}$$

and write our metric as

$$g_{rr} = 1 + \frac{2MG}{r} + \frac{|qp'|}{(4\pi)^2}\frac{1}{r^2}$$

Horizons are at $g_{rr} = 0$, a quadratic polynomial. Roots must be real, so discriminant positive:

Cosmic censorship bound

$$(MG)^2 \geq \frac{|qp'|}{(4\pi)^2} .$$

Cosmic censorship for the dilaton BH, *cont.*

Recall BPS or positivity bound:

$$M \geq |\mathcal{Z}_\infty| = (8\pi G)^{-1}\left(|q|e^{\phi} + |p'|e^{-\phi}\right) .$$

Square it:

$$M^2 \geq (8\pi G)^{-2}\left(|q|^2 e^{2\phi} + |p'|^2 e^{-2\phi} + 2|qp'|\right) .$$

But the gradient flow eqs tell us that $|\mathcal{Z}_\infty| \geq |\mathcal{Z}_{\text{horizon}}|$ (the fixed point = minimum was at the horizon). At the horizon, the attractor mechanism implied that

$$\left(e^{-2\phi}\right)\Big|_{\text{horizon}} = \left|\frac{q}{p'}\right|$$

Substitute above:

$$M^2 \geq (8\pi G)^{-2} 4|qp'| \quad \Leftrightarrow \quad (MG)^2 \geq \frac{|qp'|}{(4\pi)^2} .$$

BPS solutions and Killing spinors

How to find susy solutions (BPS solutions) of sugra? Look at susy transformations of bosons $B(x)$ and fermions $F(x)$:

$$\delta_\epsilon B(x) = \bar{\epsilon}(x) f(B)\ F(x) + (\text{fermions})\,, \quad \delta_\epsilon F(x) = g(B)\epsilon(x) + (\text{fermions})\,.$$

In classical solutions, fermions vanish. Hence solution is invariant under susy iff

$$\delta_\epsilon F(x)|_{\text{lin}} \equiv g(B)\epsilon(x) = 0\,.$$

Solutions give *Killing spinors*: they are the parameters of preserved susy. If there are n_Q independent solutions, the solution is $\frac{n_Q}{\mathcal{N}}$-BPS.

For the dilaton BH, we need terminology from $\mathcal{N} = 2$ sugra: use alternative definition of BPS via *central charge*.

Area of supersymmetric black holes

The area of a submanifold: $A = \int \sqrt{h}\,\mathrm{d}^{\mathrm{k}}\xi$: for a 2-sphere (like the horizon): $t = T, r = R$ so $\xi^1 = \theta$, $xi^2 = \varphi$, and line element becomes

$$\mathrm{d}s^2 = e^{-2U} R^2 \left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right) .$$

Therefore, area of 2-sphere with radius R is $A(R) = 4\pi e^{-2U} R^2$.

Now use $\tau = 1/r$. For $\tau \to \infty$ (horizon), the limit of $A(\tau)$ is undefined.

Solution: use l'Hôpitals' rule twice. This will bring a factor $\dot{U}$, through which central charge appears (recall: $\dot{U} = -Ge^{U}\mathcal{Z}$).

The end result is:

$$A = 4\pi G^2 |\mathcal{Z}|^2_{\text{min}} .$$

Actions

The action for the dilaton black hole:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4 x \sqrt{-g} \left[R - 2\partial^\mu \phi \partial_\mu \phi - \frac{1}{2} e^{-2\phi} \left(F^{\mu\nu} F_{\mu\nu} + F'^{\mu\nu} F'_{\mu\nu} \right) \right] ,$$

The most general action with n_V vector multiplets:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4 x \big[\sqrt{-g} \left(R - 2 g^{\mu\nu} \mathcal{K}_{\alpha\bar{\beta}} \partial_\mu z^\alpha \partial_\nu \overline{z}^{\overline{\beta}} + \tfrac{1}{2} \operatorname{Im}(\mathcal{N}_{IJ}) F^I_{\mu\nu} F^{\mu\nu J} \right) - \tfrac{1}{4} \operatorname{Re}(\mathcal{N}_{IJ}) \varepsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} \big] .$$