Holographic RG flows in gauged supergravity

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Under supervision of Nikolay Bobev

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QFTs at strong coupling

QFT is a cornerstone of modern physics (QCD, condensed matter).
For example: Standard Model, has three gauge coupling constants g.

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Needs weak coupling: $g \ll 1$.

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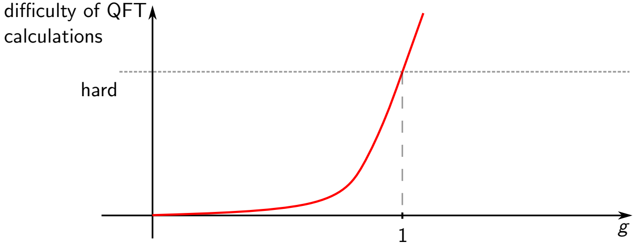
BUT: What about strong coupling $g \gg 1$? (AdS/CFT!)



Running coupling constants

QFTs have running couplings $g(\mu)$, dictated by the RG equation:

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Goal of the thesis

Demonstrate that AdS/CFT provides a window into strongly coupled phenomena in QFT, with RG flows as example.

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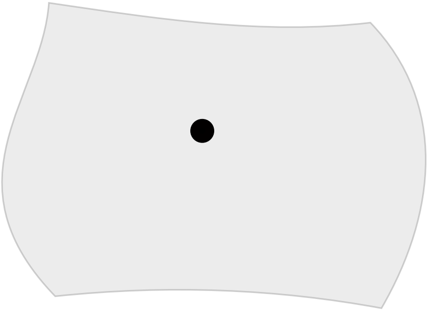
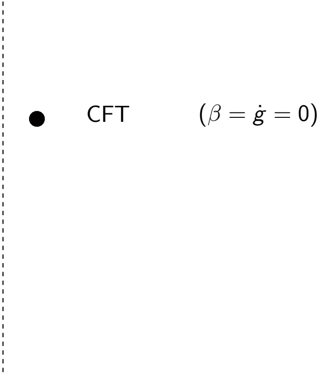
   Operators have a scaling dimension Δ (behaviour under $x \to \lambda x$).





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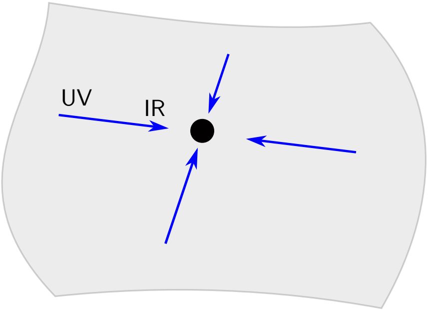

- ● CFT $(\beta = \dot{g} = 0)$
- $\longrightarrow$ irrelevant $(\Delta > d)$

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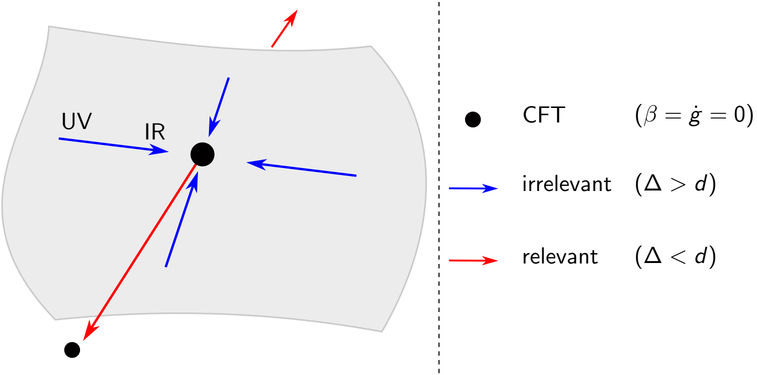

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Gauge/gravity duality

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String theory leads to a *remarkable* duality:, the gauge/gravity duality: (= AdS/CFT correspondence = holography).

Gauge QFT in d dimensions $\leftrightarrow$ Gravity theory in $(d+1)$ dimensions

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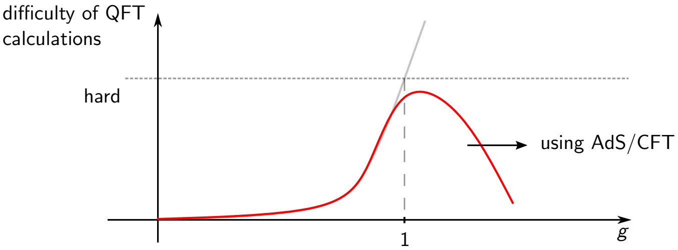
Gauge QFT in d dimensions $\leftrightarrow$ Gravity theory in $(d+1)$ dimensions

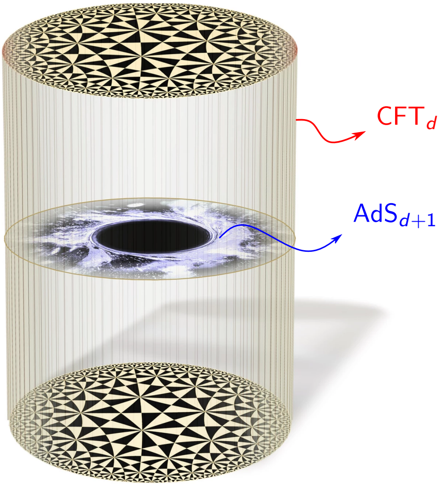
Exciting feature: is a strong/weak coupling duality!



Holographic dictionary

Correspondence between fields, operators, observables...

1. $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$



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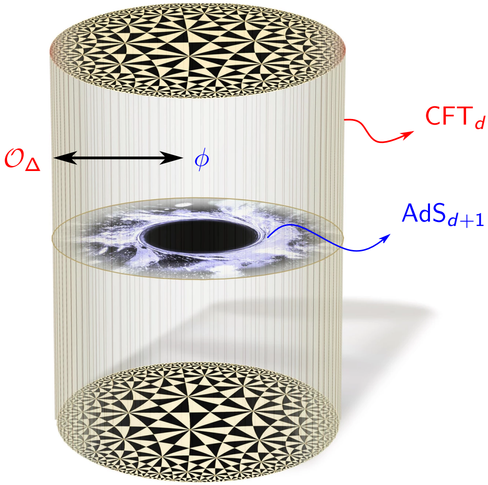

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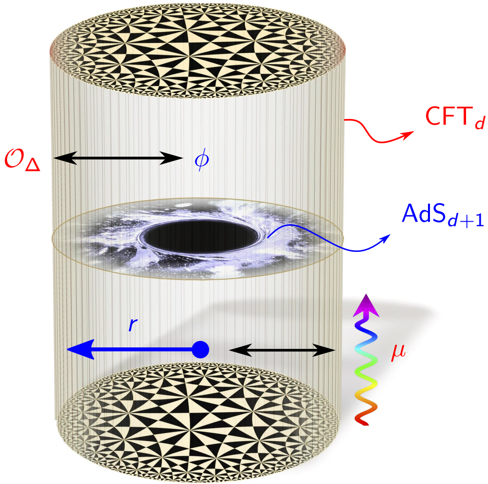
3. $r \leftrightarrow$ RG scale μ ('Holography')



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Holographic RG flows

To translate "RG flows" using the dictionary: (dynamical systems!)

	Gauge	$\leftrightarrow$	**Gravity**
Fixed points	CFT	$\leftrightarrow$	AdS
Field-operator	$\mathcal{O}_\Delta$	$\leftrightarrow$	ϕ
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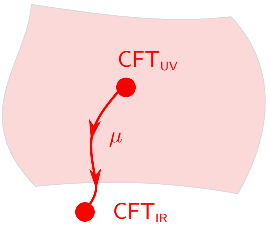

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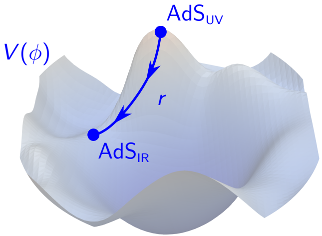

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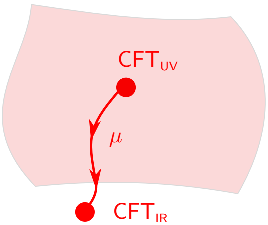

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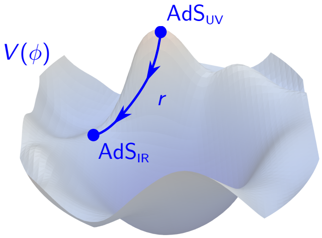

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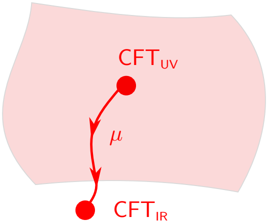
Solutions	RG flow	$\leftrightarrow$	Domain walls



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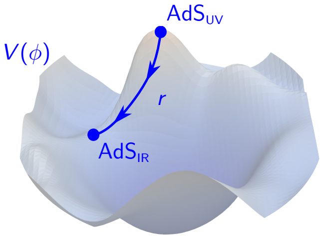

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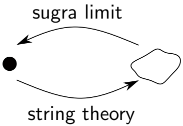

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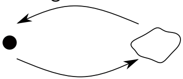

Supergravity = GR + supersymmetry.

We study maximally supersymmetric ($\mathcal{N} = 8$) 4D supergravity. We have:

- 70 spin-0 scalar fields ϕ
- 28 spin-1 vector fields $\rightarrow$ gauge group G
- 1 spin-2 space-time metric $g_{\mu\nu}$
- 8 spin-3/2 gravitini
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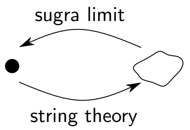

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Gauged supergravity

1. We study 4 theories (4 gauge groups) which embed into string theory. Each has a different potential $V_G(\phi)$.

$\mathbf{G = SO(8)}$ ⟶

$G = \mathrm{ISO}(7)$ ⟶

$G = [\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes \mathbb{R}^{12}$ ⟶

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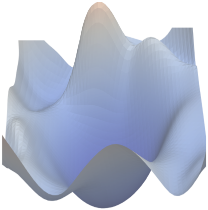

70 scalars

Gauged supergravity

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2. Hard $\rightarrow$ restrict to 14 scalar fields.

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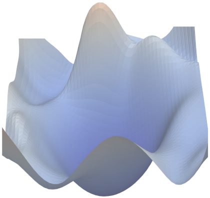
70 scalars
complicated



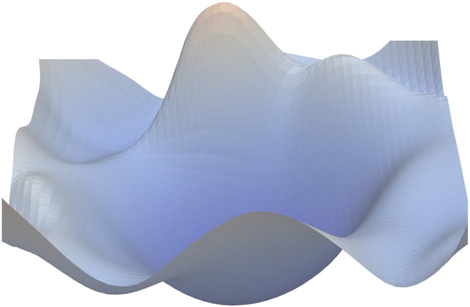
$\xrightarrow{\mathbb{Z}_2^3\text{-truncation}}$

$\mathcal{N} = 1, d = 4$ with
7 chiral multiplets

14 scalars
computable

Numerical methods

$V_G(\phi)$ depends on 14 real variables: use numerical methods.



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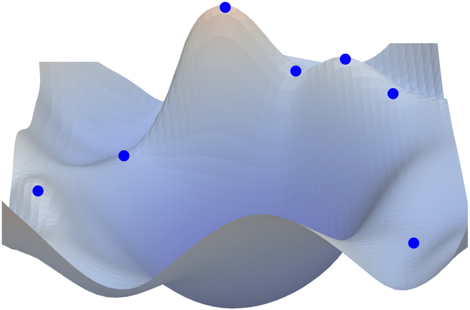

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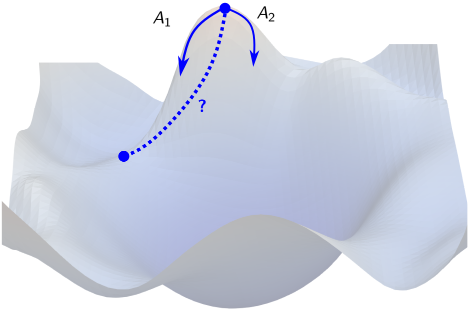
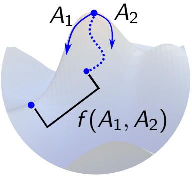
BUT: IC of flow solution depends on initial 'velocities' $A_1, \ldots, A_n$
Have to be fine-tuned: extremely complicated!!!



Step descent algorithm

OWN, NEW ALGORITHM!

Inspired by machine learning: minimize loss function $f(A_1, \ldots, A_n)$. Ours: distance between flow solution and 'target' AdS.



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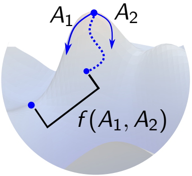

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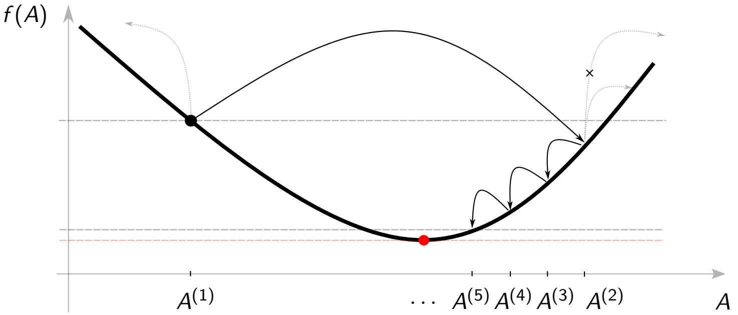

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A window into strongly coupled QFTs

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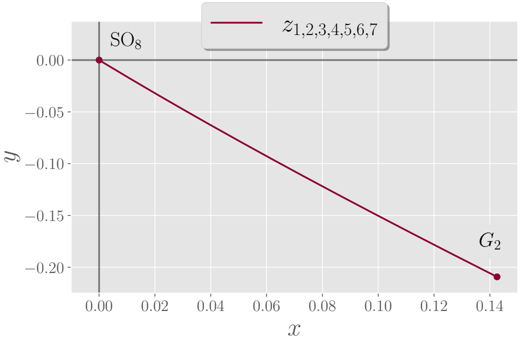
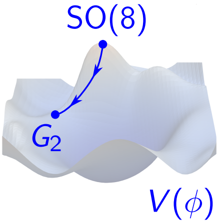
$$\mathcal{N} = 8, G = \mathrm{SO}(8) \ \rightarrow \ \mathcal{N} = 1, G = G_2 \,.$$



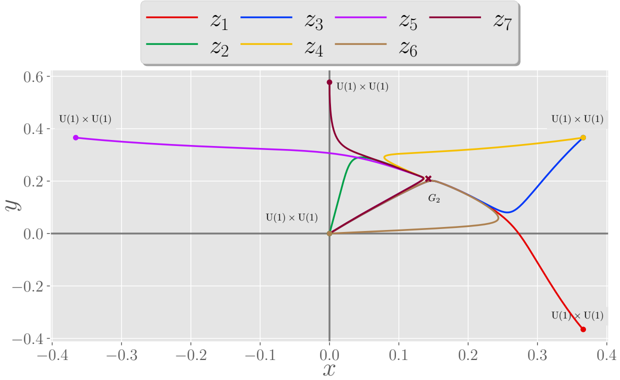


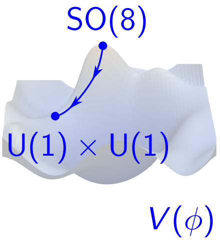
A rich web of holographic RG flows

NEW RESULT!

Within the 14 scalar truncation, we observe a rich web of RG flows!

$$\mathcal{N}=8, G=\mathrm{SO}(8) \ \to \mathcal{N}=1, G=G_2 \xrightarrow{\text{NEW}} \mathcal{N}=1, G=\mathrm{U}(1)\times\mathrm{U}(1)\,.$$





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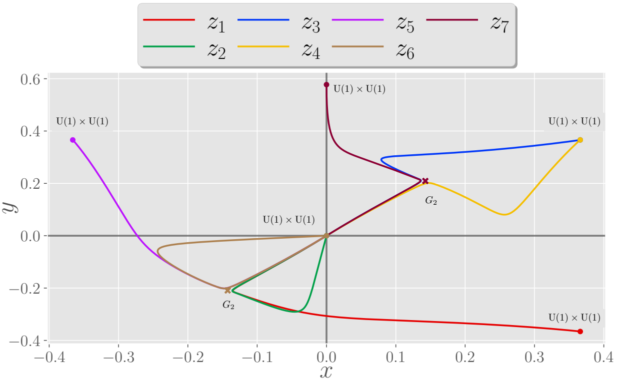

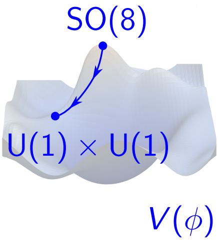

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Outlook & conclusion

- AdS/CFT provides a window into strongly coupled QFTs
- Holography allows us to study RG flows non-perturbatively
- AdS/QCD? AdS/CMT?
- Machine learning meets theoretical physics

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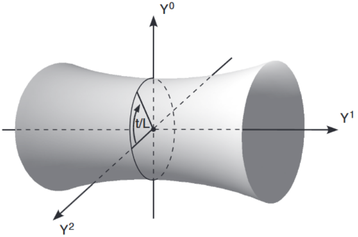
AdS space-times

Lorentzian hyperboloid: embedding into $\mathbb{R}^{2,d-1}$ with $(- + + \cdots + -)$:

$$\sum_{i=1}^{d-1} X_i^2 - X_0^2 - X_d^2 = -L^2 \,,$$

$\mathrm{SO}(2, d-1)$ invariance manifest. Metric:

$$\mathrm{d}s^2 = L^2 \left[\frac{\mathrm{d}r^2}{r^2} + r^2 \eta_{\mu\nu} \, \mathrm{d}x^\mu \, \mathrm{d}x^\nu \right] \,,$$



Conformal algebra

1. Translations: d generators P_μ,
2. Lorentz transformations: $d(d-1)/2$ generators $M_{\mu\nu}$,
3. Scale transformations: $x^\mu \to \lambda x^\mu$, with 1 generator D called the dilatation operator
4. Special conformal transformations:

$$x^\mu \to \frac{x^\mu + a^\mu x^2}{1 + 2x^\nu a_\nu + a^2 x^2}\,,$$

   with d generators K_μ.

Scaling dimensions of operators are defined by $[D, \mathcal{O}_\Delta] = \Delta \mathcal{O}_\Delta$.

They are bounded:

$$\Delta \geq \frac{d-2}{2}\,.$$

Conformal field theories

Correlation functions are simplified:

$$\langle \mathcal{O}_{\Delta_1}(x)\mathcal{O}_{\Delta_2}(y)\rangle = \frac{\delta_{\Delta_1,\Delta_2}}{|x-y|^{2\Delta_1}}\,,$$

$$\langle \mathcal{O}_{\Delta_1}(x)\mathcal{O}_{\Delta_2}(y)\mathcal{O}_{\Delta_3}(z)\rangle = \frac{c_{123}}{|x-y|^{\Delta_{12}}|y-z|^{\Delta_{23}}|x-z|^{\Delta_{13}}}\,,$$

Higher order: operator product expansion:

$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) \xrightarrow{x_1\to x_2} \sum_i C_{12}^{(i)}(x_1,x_2)\mathcal{O}_i(x_2)\,.$$

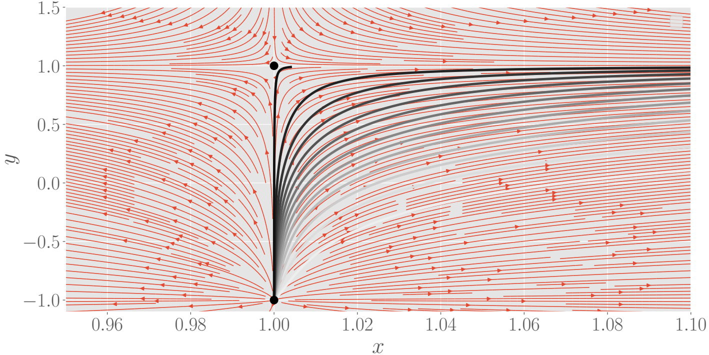
Finely-tuned flows

For the system

$$\begin{cases} \dot{x} &= -1 + x^3\,, \\ \dot{y} &= 1 - y^2\,. \end{cases}$$

Jacobian matrix:

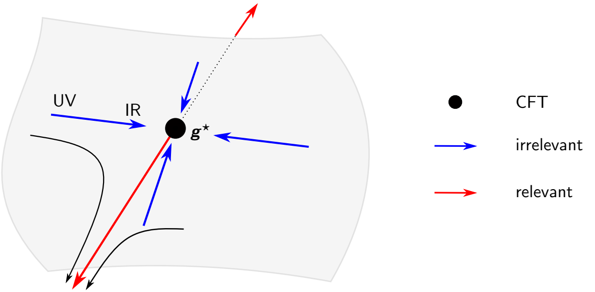
$$\mathcal{J}(x,y) = \begin{pmatrix} 3x^2 & 0 \\ 0 & -2y \end{pmatrix}\,.$$



RG flows and scaling dimensions

$$\boldsymbol{g}(\Lambda) = \boldsymbol{g}^\star + \sum_{i=1}^{n} A_i \boldsymbol{v}_i \left(\frac{\Lambda}{\Lambda_0}\right)^{\Delta_i - d} , \tag{8.1}$$

- $\Delta > d$: irrelevant
- $\Delta < d$: relevant



Gradient flow equations

The scalar potential $V(\phi)$ (1 scalar field) can be written as

$$V(\phi) = \frac{1}{2}\left(\frac{\mathrm{d}W}{\mathrm{d}\phi}\right)^2 - \frac{d}{2(d-1)}W^2\,,$$

where $W(\phi)$ is called the superpotential.

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where $W(\phi)$ is called the superpotential.

The gradient flow equation solving the gravity equations of motion is

$$\phi' = \frac{\mathrm{d}W}{\mathrm{d}\phi}\,,$$

with $' = \frac{\partial}{\partial r}$.

Gradient flow equations

The scalar potential $V(\phi)$ (1 scalar field) can be written as

$$V(\phi) = \frac{1}{2}\left(\frac{\mathrm{d}W}{\mathrm{d}\phi}\right)^2 - \frac{d}{2(d-1)}W^2\,,$$

where $W(\phi)$ is called the superpotential.

The gradient flow equation solving the gravity equations of motion is

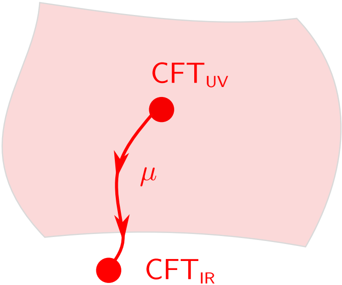
$$\phi' = \frac{\mathrm{d}W}{\mathrm{d}\phi}\,,$$

with $' = \frac{\partial}{\partial r}$.

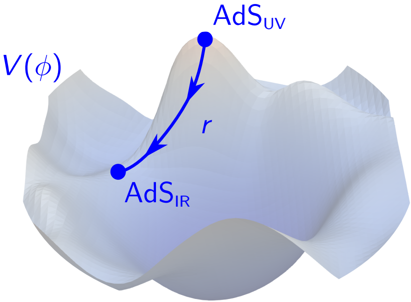
Resembles the RG equation $\dot{g} = \beta(g)$.
The "extra" AdS_{d+1} coordinate r corresponds to the RG scale μ in CFT_d.

The gauge/gravity duality implies that domain wall solutions are dual to RG flows, based on

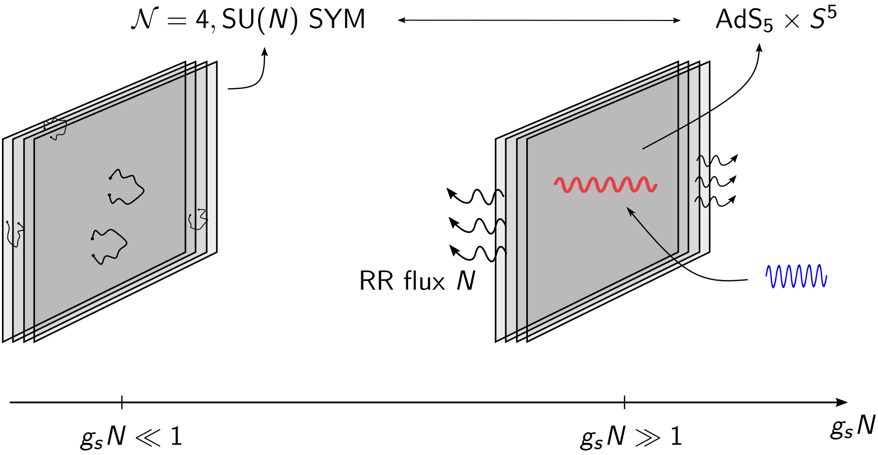
$$\Delta(\Delta - d) = m_\phi^2 L^2\,, \quad V(\phi) \approx V(\phi^\star) + \tfrac{1}{2} m_\phi^2 \phi^2$$



$\leftrightarrow$



AdS/CFT original formulation



Type IIB string theory on $\mathrm{AdS}_5 \times S^5 \leftrightarrow \mathcal{N} = 4, d = 4, G = \mathrm{SU}(N)$ SYM

Holographic dictionary

The precise relation for $\mathcal{O}_\Delta \leftrightarrow \phi$ is:

$$\Delta(\Delta - d) = m_\phi^2 L^2$$

L from the AdS metric:

$$\mathrm{d}s^2 = L^2 \left[\frac{\mathrm{d}r^2}{r^2} + r^2 \eta_{\mu\nu} \, \mathrm{d}x^\mu \, \mathrm{d}x^\nu \right] ,$$

Squared masses can be negative: BF bound

$$-\frac{d^2}{4} \leq m^2 L^2 \, .$$

For some range: both $\Delta_\pm$ above the unitarity bound $\Delta \geq \frac{d-2}{2}$.
Susy selects the correct root.

Domain wall solutions

Domain wall ansatz

$$\mathrm{d}s^2 = e^{2r/L}\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + \mathrm{d}r^2\,, \quad \phi = \phi(r)$$

EoM:

$$\phi'' + dA'\phi' = \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}\,,$$

$$(A')^2 = \frac{1}{d(d-1)}\left((\phi')^2 - 2V(\phi)\right)$$

Introduce

$$V(\phi) = \frac{1}{2}\left(\frac{\mathrm{d}W}{\mathrm{d}\phi}\right)^2 - \frac{d}{2(d-1)}W^2\,,$$

The gradient flow eqs are:

$$\phi' = \frac{\mathrm{d}W}{\mathrm{d}\phi}\,, \quad A' = -\frac{1}{d-1}W$$

Gauged supergravities and dual field theories